

Compound Interest

Compound interest differs from Simple interest as the interests earned in the previous years considered as principal for the next years. Observe the table

Years	1	2	3	4
SI	10	10	10	10
CI	10	11	12.1	13.31

10 + 1

10 + 1 +
1 + 0.1

10 + 1 +
1 + 1 + 0.1
+ 0.1 + 0.1
+ 0.01

From the above table, we can see that in Simple interest case, interests earned in each year is same, but in the compound interest, except for the first year, interests are different. This is due to the fact that in the CI case interest earned in the 1st year is considered as principal and interest will be calculated on this too. In the first box 10+1 can be explained as 10 is the interest on the principal, 1 is the interest on the first year interest. For the 2nd box, 10 is the interest on principal, 1 is interest on first year interest, another 1 is interest on second year 10, and 0.1 is the interest on 1.

There is no direct formula for Compound interest but $P(1 + \frac{R}{100})^n$ gives us the Amount which includes principal as well as interest. So to find the interest we need to subtract principal from the amount.

$$CI = A - P$$

Compound interest through Pascal Triangle:

Find the compound interest on Rs.6000 at the rate 10% per annum for 3 years.

$$\text{Formula: } A = P(1 + \frac{R}{100})^n = 6000(1 + \frac{10}{100})^3$$

$$\Rightarrow 6000 \times (\frac{11}{10})^3 = 7986$$

$$\text{Compound Interest} = 7986 - 6000 = 1986$$

Alternate method:

We can solve compound interest problems easily with the use of pascal triangle.

			1		
		1		1	
	1		2		1
1		3		3	

To solve above problem, we need to identify the coefficients where 3 appears first time in a row. We know that in 4th row 3 appears. We need to take coefficients excluding 1. i.e., 3, 3, 1. Now

$$CI = 3 \times (10\% \text{ of } 6000) + 3 \times (10\% \text{ of } ()) + 1 \times ()$$

$$CI = 3 \times (10\% \text{ of } 6000) + 3 \times (10\% \text{ of } 600) + 1 \times (10\% \text{ of } 60) = 1986$$

To find the difference between compound interest and simple interest for two years :

$$\text{Compound interest for two years} = P(1+r)^2 - P$$

$$\text{Simple interest for two years} = P \times r \times 2 = 2Pr$$

$$\text{Here, } r = \frac{\text{Rate of interest}}{100}$$

$$= [P(1+r)^2 - P] - (2Pr)$$

$$= P(1 + 2r + r^2) - P - (2Pr)$$

$$= P + 2Pr + Pr^2 - P - 2Pr = Pr^2$$

$$\text{Hence, difference between compound interest and simple interest @ } r \% \text{ p.a. for two years} = P(r^2)$$

$$\text{Difference} = \text{Principal} \times \left(\frac{\text{Rate}}{100} \right)^2$$

To find the difference between Compound Interest and Simple interest, when Simple Interest for 2 years is given:

$$\text{Difference between compound interest and simple interest for 2 years} = P(r^2) = Pr(r)$$

$$\text{But, } Pr = \text{Principal} \times \frac{\text{Rate}}{100} = \text{Simple Interest for one year}$$

$$= \frac{1}{2} \times \text{Total Simple Interest}$$

Therefore, Difference between compound interest and simple interest for 2 years

$$= Pr(r) = \frac{1}{2} \times \text{Simple Interest} \times (r)$$

$$= \frac{1}{2} r \text{ of Simple Interest}$$

$$\text{For 2 years: } \frac{1}{2} r \text{ of simple interest}$$

Alternate Method:

Without the above derivation, we can calculate the difference by using the reasoning given below.

We know that Compound interest for first year = Simple Interest for one year

Compound Interest for second year = Simple Interest for one year + Interest on Simple Interest for the first year
(have a look at the table)

Therefore Difference = Interest on Simple interest of first year

To find the difference between compound interest and simple interest for three years :

$$\begin{aligned} & [P(1+r)^3 - P] - P \times 3 \times r \\ \Rightarrow & [P(1+3r+3r^2+r^3) - P] - P \times 3 \times r \\ \Rightarrow & 3Pr^2 + Pr^3 \\ \Rightarrow & Pr(3r+r^2) \end{aligned}$$

$$\text{Difference for 3 years} = Pr^2 (3 + r)$$

To find the difference between Compound Interest and Simple interest, when Simple Interest for 3 years is given:

For 3 years: $\frac{1}{3} r (3 + r)$ of simple interest

Note: 'r' is $\frac{\text{Rate of interest}}{100}$

Difference between compound interest and simple interest for 3 years = $Pr^2 (3 + r) = Pr (r) (3 + r)$

Now, $Pr = \text{Principal} \times \frac{\text{Rate}}{100} = \text{Simple Interest for one year}$

$$= \frac{1}{3} \times \text{Total Simple Interest}$$

Therefore, Difference between compound interest and simple interest for 3 years

$$= Pr (r) (3 + r) = \frac{1}{3} \times \text{Simple Interest} \times (r) (3 + r)$$

$$= \frac{1}{3} r (3 + r) \text{ of Simple Interest}$$

To find simple interest when compound interest is given? If difference between compound interest and simple interest is $\frac{1}{x}$ of Simple interest Then, difference between compound interest and simple interest is $\frac{1}{x-1}$ of Compound Interest.

Hint: Compound Interest is more than Simple Interest.

And, Compound Interest = Simple Interest + Difference

Therefore, If we shift from Simple Interest to Compound Interest, smaller fraction is required and vice-versa.

To find Compound Interest for two years, when Simple interest is given:

Compound Interest = Total Simple interest + (Simple interest for one year x Rate of Interest)

Practice problems

1. Find the amount for Rs. 6000 at 10% per annum, compounded semi-annually for 2 years. Here $n = 2 \text{ years} \times 2 = 4$

periods

$$\text{Similarly, } R = \frac{10}{2} = 5\% \text{ (for half year)}$$

$$P = 6,000$$

$$A = 6,000 \left(1 + \frac{5}{100}\right)^4 = \text{Rs. } 7,293$$

$$\text{Interest} = \text{Rs. } 7293 - 6000 = 1293$$

Alternate method:

Using pascal triangle: the coefficients are 4, 6, 4, 1 for 4 years.

$$CI = 4 \times (5\% \text{ of } 6000) + 6 \times (5\% \text{ of } ()) + 4 \times (5\% \text{ of } ()) + 1 \times ()$$

$$= 4 \times (300) + 6 \times (15) + 4 \times (.75) + () = 1293$$

We can stop after using 3 coefficients as the 4th coefficient multiple is too small and may not cause any big difference in our answer.

2. The difference between the CI and SI on a certain amount at 10% per annum for 2 years, compounded annually is Rs. 372. Find the principal.

Let the principal be a.

$$SI = \frac{a \times 2 \times 10}{100} = \frac{a}{5} \quad \text{and} \quad CI = \text{Amount} - a = a \left(1 + \frac{10}{100}\right)^2 - a = \frac{21}{100} \times a$$

$$CI - SI = \text{Rs. } 372$$

$$\frac{21}{100} \times a - \frac{a}{5} = \text{Rs. } 372$$

$$a = \text{Rs. } 37,200$$

Alternate method:

We know that the difference in interest comes from second year. Assume principal is Rs.100 then Interests are calculated as below.

Rs. 100	SI	CI
1st year	10	10
2nd year	10	10 + 1
	20	21

Now the difference is Rs.1 but actually it was given as Rs.372. If principal is Rs.100 difference is Rs.1, what is the principal if difference has to be Rs.372

$$\Rightarrow \frac{372}{1} \times 100 = 37200$$

Alternate method:

The above problem has an alternate method. You need to understand the fact that for 1st period, SI = CI.

The difference between the values of CI and SI is because of accumulated interest building on interest which is reinvested. Therefore, for period 2, the difference between CI and SI is the interest for 1 period on the interest of period 1.

In the above example, the difference being 372 is the interest generated on interest for period 1 on the principal.

$$\text{Interest for period 1} = \text{Rs. } 372 \times \frac{100}{10} = \text{Rs. } 3,720$$

$$\text{Therefore, principal} = \text{Rs. } 3,720 \times \frac{100}{10} = \text{Rs. } 37,200.$$

3. Find compound interest on Rs. 10000 at 10% p.a. for 4 years, if interest is compounded annually.

$$\text{Amount} = \text{Rs. } 10000 \times \left(\frac{11}{10}\right)^4 = 14641$$

$$\text{Therefore, Compound interest} = \text{Rs. } 14641 - \text{Rs. } 10000 = \text{Rs. } 4641$$

Note: Steps for calculation of 11^4 :

$$11 \times 11 = 121; 121 \times 11 = 1331; 1331 \times 11 = 14641$$

Approximate Method:

We know that,

Compound interest = Simple interest + Interest on simple interest

$$\text{Simple interest} = 10000 \times \frac{1}{10} \times 4 = \text{Rs. } 4000$$

$$\text{Therefore, Simple interest for one year} = \text{Rs. } 4000 \div 4 = \text{Rs. } 1000.$$

Therefore, Interest on interest

$$= 0 + 10\% \text{ of Rs. } 1000 + 10\% \text{ of Rs. } 2000 + 10\% \text{ of Rs. } 3000 + \text{interest on interest}$$

$$= 0 + 100 + 200 + 300 + \text{interest on this amount}$$

$$= 600 + \text{interest on this amount}$$

$$\text{Therefore, Compound interest} = \text{Rs. } 4000 + \text{Rs. } 600 + \text{interest on Rs. } 600 = \text{Rs. } 4600 + \text{Interest on Rs. } 600$$

Out of given options, amount nearest to it is Rs. 4641.

4. If a certain sum of money invested at a certain rate of compound interest doubles in 5 years. In how many years will it become 4 times?.

$$\text{Since, } 2^2 = 4.$$

Therefore, The amount will become 4 times in $2 \times 5 = 10$ years.

5. At what rate per cent of compound interest, a sum of Rs. 2000 will amount to Rs. 2662 in 3 years?

We know that, $(1 + \frac{\text{Rate}}{100})^{\text{Time}} = \frac{\text{Amount}}{\text{Principal}}$

$$\text{Therefore, } (1 + r)^3 = \frac{2662}{2000} = \frac{1331}{1000} = \left(\frac{11}{10}\right)^3$$

$$\text{Therefore, } 1 + r = \frac{11}{10}$$

$$\text{Therefore, } r = \frac{11}{10} - 1 = \frac{1}{10} = 10\%$$

6. A man invested Rs. 16000 at compound interest for 3 years, interest compounded annually. If he got Rs. 18522 at the end of 3 years, what is rate of interest?

$$\text{Here, } (1 + r)^3 = \frac{18522}{16000} = \frac{9261}{8000} = \left(\frac{21}{20}\right)^3 = \left(1 + \frac{1}{20}\right)^3$$

$$\text{Therefore, Rate of interest} = \frac{1}{20} = 5\%$$

'Approximate Method':

Compound interest = Rs. 18522 - Rs. 16000 = Rs. 2522

Let, the amount is invested at 1% p.a. simple interest.

Then, simple interest of 3 years = $16000 \times 1\% \times 3$ = Rs. 480

$$\text{Therefore, Rate of interest} = \frac{2522}{480} = 5 + (\text{Remainder is Rs. 122})$$

We know that compound interest is more than simple interest.

Note: If Rate is 6%, then simple interest = 480×6 = 2880, which is more than the given compound interest which is not possible.

Therefore, Rate of interest $\geq 6\%$ is not possible.

Therefore, Rate of interest is 5% p.a.

7. A sum of money amounts to Rs. 2880 in 2 years and 3456 in 3 years at compound interest. Find the sum.

Rs. 2880 amounts to Rs. 3456 in one year.

The sum amounts to $\frac{3456}{2880} = \frac{6}{5}$ times of itself

$$\text{Therefore, Principal} = 2880 \div \left(\frac{6}{5}\right)^2 = 2880 \times \frac{5}{6} \times \frac{5}{6} = \text{Rs. 2000}$$

8. A man borrows Rs. 2100 and undertakes to pay back with compound interest @ 10% p.a. in 2 equal yearly installments at the end of first and second year. What is the amount of each installment? Here, $(1 + r) = 1 +$

$$\frac{1}{10} = \frac{11}{10}$$

$$\text{Ratio of principals of two instalments} = 1 : \frac{10}{11} = 11 : 10$$

$$\text{Sum of ratios} = 11 + 10 = 21$$

$$\text{Therefore, Principal of first instalment} = 2100 \times \frac{11}{21} = \text{Rs. 1100}$$

Therefore, Instalment = Principal of first instalment $\times (1 + r)$

$$= 1100 \times \frac{11}{10} = \text{Rs. 1210}$$

9. A man borrows Rs. 820 and undertakes to pay back with compound interest @ 5% p.a. in 2 equal yearly instalments at the end of first and second year. What is the amount of each installment?

$$\text{Here, } (1 + r) = 1 + \frac{1}{20} = \frac{21}{20}$$

$$\text{Ratio of principals of two instalments} = 1 : \frac{20}{21} = 21 : 20$$

$$\text{Sum of ratios} = 21 + 20 = 41$$

$$\text{Therefore, Principal of first instalment} = \frac{21}{41} \times 820 = \text{Rs. } 420$$

$$\text{Therefore, Instalment} = \text{Principal of first instalment} \times (1 + r) = 420 \times \frac{21}{20} = \text{Rs. } 441$$

10. A man borrows Rs. 1820 and undertakes to pay back with compound interest @ 20% p.a. in 3 equal yearly installments at the end of first, second and third years. What is the amount of each installment?

$$\text{Here, } (1 + r) = 1 + \frac{1}{5} = \frac{6}{5}$$

$$\text{Ratio of principals for three years} = 1 : \frac{5}{6} : \left(\frac{5}{6}\right)^2$$

$$= 6^2 : 6 \times 5 : 5^2 \text{ (On multiplying each ratio by } 6^2 \text{)}$$

$$= 36 : 30 : 25$$

$$\text{Sum of the ratios} = 36 + 30 + 25 = 91$$

$$\text{Therefore, Principal of first installment} = \frac{36}{91} \times 1820 = \text{Rs. } 720$$

$$\text{Therefore, Installment} = \text{Principal of first installment} \times (1 + r) = 720 \times \frac{6}{5} = \text{Rs. } 864$$

11. A certain sum is to be divided between A and B so that after 5 years the amount received by A is equal to the amount received by B after 7 years. The rate of interest is 10%, interest compounded annually. Find the ratio of amounts invested by them.

Let the sum (principal) received by A and B are x and y.

$$(1 + r) = 1 + \frac{1}{10} = \frac{11}{10}$$

$$\text{Then, } \frac{x}{y} = \left(\frac{11}{10}\right)^{7-5} = \left(\frac{11}{10}\right)^2 = \frac{121}{100}$$

Hence, the ratio in which the sum is divided = 121 : 100.

12. A father wants to divide Rs. 5100 between his two sons, Mohan and Sohan who are 23 and 24 at present. Divide the amount in such a way that if their shares are invested at compound interest @ 4% p.a. they will receive equal amount on attaining the age of 26 years. Find Mohan's share.

Let, Mohan and Sohan receives Rs. x and Rs. y respectively at present.

$$(1 + r) = 1 + \frac{1}{25} = \frac{26}{25}$$

$$\text{Then, } \frac{x}{y} = \left(\frac{26}{25}\right)^{2-3} = \left(\frac{26}{25}\right)^{-1} = \frac{25}{26}$$

$$\text{Therefore, Mohan's share} = \frac{25}{51} \times \text{Rs. } 5100 = \text{Rs. } 2500$$

13. Find the difference between Compound Interest and Simple Interest on Rs. 4000 for 1 year at 10% p.a., if the

interest is compounded half-yearly.

Since, interest is compounded half-yearly. Therefore, Rate of interest is halved and time is doubled. Therefore, Rate

$$= \frac{10}{2} \% = 5\% = \frac{1}{20}$$

And, Time = $2 \times 1 = 2$ half-years.

Therefore, Difference between Compound Interest and Simple Interest = Rs. $4000 \times \frac{1}{20} \times \frac{1}{20} = \text{Rs. } 10$

14. Find the difference between Compounded Interest and Simple Interest on Rs. 1000 for 3 years at 10% p.a., if interest is compounded annually.

$$\begin{aligned} \text{Difference between Compound Interest and Simple Interest for 3 years} &= Pr^2 (3 + r) = \text{Rs. } 1000 \times \frac{1}{10} \times \frac{1}{10} \times \\ (3 + \frac{1}{10}) &= \text{Rs. } 31 \end{aligned}$$

15. Find the difference between Compound Interest and Simple Interest on Rs. 10000 for 4 years at 10% p.a., if interest is compounded annually.

$$\begin{aligned} \text{Difference between Compound Interest and Simple Interest for 4 years} &= Pr^2 (6 + 4r + r^2) = 10000 \times \frac{1}{10} \times \frac{1}{10} \times \\ (6 + \frac{4}{10} + \frac{1}{100}) &= 10000 \times \frac{1}{100} \times \frac{641}{100} = \text{Rs. } 641 \end{aligned}$$

16. If Compound Interest on a certain sum for 2 years @ 5% p.a. is Rs. 328, the Simple interest will be ?

Suppose, Compound Interest for first year = Rs. 100

Then, Compound Interest for second year = Rs. 105

Total Compound Interest for two years = (Rs. 100 + Rs. 105) = Rs. 205

And Simple Interest for two years = $2 \times \text{Rs. } 100 = \text{Rs. } 200$

If Compound Interest is Rs. 205, Simple Interest = Rs. 200

$$\begin{aligned} \text{If Compound Interest is Rs. 328, Simple Interest} &= \text{Rs. } 328 \times \frac{200}{205} \\ &= \text{Rs. } 320 \end{aligned}$$

Alternative Method:

$$\text{Rate} = 5\% = \frac{1}{20}$$

Difference between Compound interest and Simple interest

$$= \frac{1}{2} \times \frac{1}{20} = \frac{1}{40} \text{ of simple interest}$$

$$= \frac{1}{41} \text{ of the compound interest} = \frac{1}{41} \times \text{Rs. } 328 = \text{Rs. } 8$$

Therefore, Simple interest = Compounded interest - Difference

$$= \text{Rs. } 328 - \text{Rs. } 8 = \text{Rs. } 320$$

17. If a certain sum of money invested at a certain rate of compound interest doubles in 6 years. In how many years will it become 8 times?

Solution:

Since, $2^3 = 8$.

Therefore, The amount will become 8 times in $3 \times 6 = 18$ years.

MCQ's

1. A sum of money becomes Rs.6690 after three years and Rs.10,035 after 6 years on compound interest. The sum is :

- a. Rs.4400
- b. Rs.4445
- c. Rs.4460
- d. Rs.4520

Correct Option: C

Explanation:

Let the sum be P.

$$\text{Then, } P \left[1 + \frac{R}{100}\right]^3 = 6690 \dots\dots(i)$$

$$\text{and } P \left[1 + \frac{R}{100}\right]^6 = 10,035 \dots\dots(ii)$$

Dividing (ii) by (i), we get

$$\left(1 + \frac{R}{100}\right)^3 = \frac{10035}{6690} = \frac{3}{2}$$

$$P = \left(6690 \times \frac{2}{3}\right) = \text{Rs.}4460$$

2. Rs.1600 at 10% per annum compound interest compound half-yearly amount to Rs.1944.81 in

- a. 2 years
- b. 3 years
- c. $1\frac{1}{2}$ years
- d. $2\frac{1}{2}$ years

Correct Option: A

Explanation:

$$1600 \left(1 + \frac{5}{100}\right)^T = 1944.81$$

$$\Rightarrow \left(\frac{21}{20}\right)^T = \frac{1944.81}{1600.00} = \frac{194481}{160000}$$

$$= \left(\frac{441}{400}\right)^2 = \left(\frac{21}{20}\right)^4$$

$T = 4$ (Half - years) or $T = 2$ years

3. The difference between simple interest and compound interest on a sum for 2 years at 8%, when the interest is compounded annually Rs.16. If the interest was compounded half-yearly, the difference in two interests would be nearly :

- a. Rs.16
- b. Rs.16.80
- c. Rs.21.85
- d. Rs.24.64

Correct Option: D

Explanation:

For 1st year, S.I = C.I

Thus, Rs.16 is the S.I on S.I for 1 year, which at 8% is thus Rs.200

i.e.S.I on the principal for 1 year is Rs.200

$$\text{Principal} = \text{Rs.} \left(\frac{100 \times 200}{8 \times 1} \right) = \text{Rs.}2500$$

Amount for 2 years, compounded half-yearly

$$= \text{Rs.} \left[2500 \times \left(1 + \frac{4}{100} \right)^4 \right] = \text{Rs.}2924.64$$

$$\text{C.I} = \text{Rs.}424.64$$

$$\text{Also, S.I} = \text{Rs.} \left(\frac{2500 \times 8 \times 2}{100} \right) = \text{Rs.}400$$

$$\text{Hence, } [(C.I)-(S.I)] = \text{Rs.} (424.64-400)=\text{Rs.}24.64$$

4. The difference in C.I and S.I for 2 years on a sum of money is Rs.160. If the S.I for 2 years be Rs.2880, the rate percent is :

- a. $5\frac{5}{9}\%$
- b. $12\frac{1}{2}\%$
- c. $11\frac{1}{9}\%$
- d. 9%

Correct Option: C

Explanation:

$$\text{S.I for 1 year} = \text{Rs.}1440$$

$$\text{S.I on Rs.}1440 \text{ for 1 year} = \text{Rs.}160$$

$$\text{Hence, rate percent} = \left(\frac{100 \times 160}{1440 \times 1} \right) = 11\frac{1}{9}\%$$

5. The value k of a machine depreciates every year at the rate of 10% on its value at the beginning of that year. If the present value of the machine is Rs.729, its worth 3 years ago was :

- a. Rs.947.10
- b. Rs.800
- c. Rs.1000
- d. Rs.750.87

Correct Option: C

Explanation:

$$P = \left(1 - \frac{10}{100}\right)^3 = 729$$

P=Rs.

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